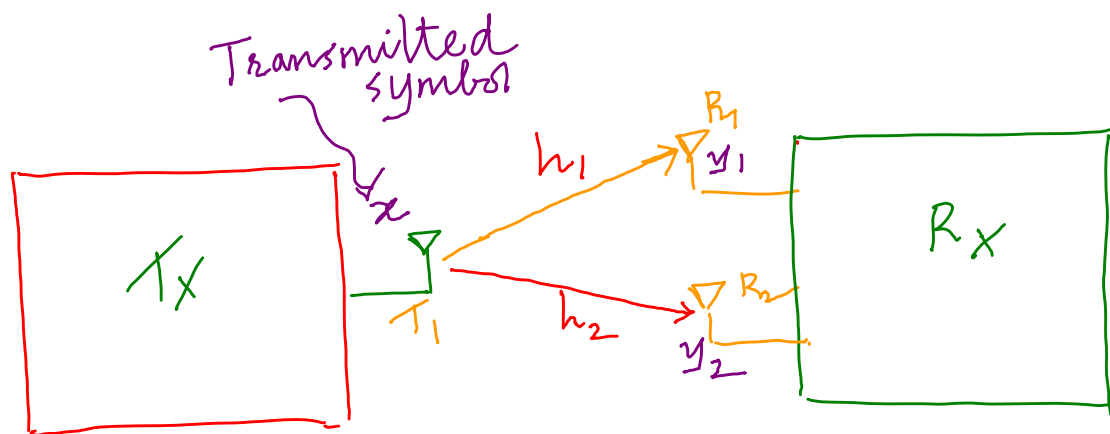


Multiple Antenna System:



$h_1 =$ Fading coeff between T_1, R_1
 $h_2 =$ Fading coeff between T_1, R_2
 $y_1 =$ Symbol received on R_1
 $y_2 =$ Symbol received on R_2

System can be modeled as,

real quantities, $y_1 = h_1 x + n_1$ ← noise at R_1
 $y_2 = h_2 x + n_2$ ← noise at R_2

n_1, n_2 are Gaussian

→ Zero mean

→ Each has power σ^2

$$E\{|n_1|^2\} = E\{|n_2|^2\} = \sigma^2$$

$$E\{n_1 n_2\} = 0 \rightarrow n_1, n_2 \text{ are uncorrelated.}$$

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}}_{\substack{\text{Received} \\ \text{vector} \\ \bar{y}}} = \underbrace{\begin{bmatrix} h_1 \\ h_2 \end{bmatrix}}_{\substack{\text{channel} \\ \text{vector} \\ \bar{h}}} x + \underbrace{\begin{bmatrix} n_1 \\ n_2 \end{bmatrix}}_{\substack{\text{Noise} \\ \text{vector} \\ \bar{n}}}$$

$$\bar{y} = \bar{h} x + \bar{n}$$

we have 2 received symbols.

y_1, y_2

Combine these 2 symbols.

$$\hat{y} = \overbrace{w_1 y_1 + w_2 y_2}^{\text{combining weights}}$$

$$= [w_1 \ w_2] \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$= \bar{w}^T \bar{y} \quad \bar{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad \bar{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$\begin{aligned}
 \tilde{y} &= \bar{w}^T(\bar{y}) \\
 &= \bar{w}^T(\mathbf{h}x + \bar{n}) \\
 &= \underbrace{\bar{w}^T \mathbf{h} x}_{\text{Signal Component}} + \underbrace{\bar{w}^T \bar{n}}_{\text{noise Component}}
 \end{aligned}$$

$$\text{SNR} = \frac{|\bar{w}^T \mathbf{h}|^2 P}{E\{|\bar{w}^T \bar{n}|^2\}} = \frac{|\bar{w}^T \mathbf{h}|^2 P}{\sigma^2 \|\bar{w}\|^2}$$

Noise:

$$\bar{w}^T \bar{n} = \begin{bmatrix} w_1 & w_2 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$= w_1 n_1 + w_2 n_2$$

$$E\left((w_1 n_1 + w_2 n_2)^2\right)$$

$$= E\left(w_1^2 n_1^2 + w_2^2 n_2^2 + 2w_1 w_2 n_1 n_2\right)$$

$$= w_1^2 \overbrace{E\{n_1^2\}}^{\sigma^2} + w_2^2 \overbrace{E\{n_2^2\}}^{\sigma^2}$$

$$+ 2w_1 w_2 \underbrace{E\{n_1 n_2\}}_0$$

$$= w_1^2 \sigma^2 + w_2^2 \sigma^2$$

$$E \{ |\bar{w}^T \bar{n}|^2 \} = w_1^2 \sigma^2 + w_2^2 \sigma^2$$

$$\text{Noise Power} = \sigma^2 (w_1^2 + w_2^2)$$

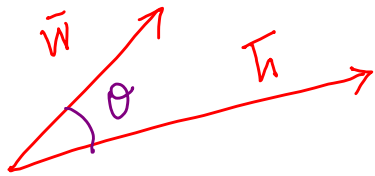
$$= \sigma^2 \|\bar{w}\|^2$$

$$\bar{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \quad \|\bar{w}\|^2 = w_1^2 + w_2^2$$

$$SNR = \frac{P |\bar{w}^T \bar{h}|^2}{\sigma^2 \|\bar{w}\|^2}$$

$$\bar{w}^T \bar{h} = [w_1 \ w_2] \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = w_1 h_1 + w_2 h_2$$

$$= \underline{\bar{w} \cdot \bar{h}}$$

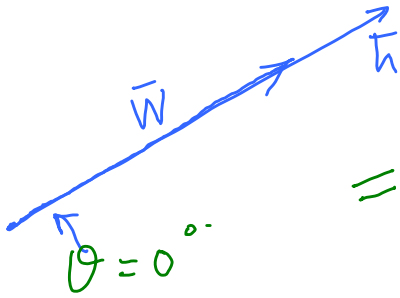


From Vector Calculus.
Dot Product between $\bar{w}, \bar{h}$

$$|\bar{w} \cdot \bar{h}| = \|\bar{w}\| \cdot \|\bar{h}\| \cos \theta$$

$$|\bar{w} \cdot \bar{h}|^2 = \|\bar{w}\|^2 \cdot \|\bar{h}\|^2 \cos^2 \theta$$

$$SNR = \frac{P \cdot \cancel{\|\bar{w}\|^2} \|\bar{h}\|^2 \cos^2 \theta}{\sigma^2 \cancel{\|\bar{w}\|^2}}$$



$$= \frac{P \cdot \|\bar{h}\|^2 \cos^2 \theta}{\sigma^2}$$

This is maximum, when $\cos^2 \theta$ is maximum.

maximum value of $\cos^2 \theta = 1$
occurs when $\theta = 0$

For SNR to be maximized,

$$\theta = 0^\circ$$

$\Rightarrow \bar{w}$ has to be along $\bar{h}$

$$\bar{w} \propto \bar{h}$$

$$SNR = \frac{P \cdot \|\bar{h}\|^2}{\sigma^2} = \frac{P(|h_1|^2 + |h_2|^2)}{\sigma^2}$$

$$\bar{w} = \frac{1}{\|\bar{h}\|} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \frac{1}{\sqrt{|h_1|^2 + |h_2|^2}} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

For complex coefficients, use $\bar{w}^H \bar{y}$

$$\bar{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$\bar{w}^H \bar{y} = [w_1^* \ w_2^*] \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

combining at receiver

$$= w_1^* y_1 + w_2^* y_2$$

$$\bar{w} = \frac{\bar{h}}{\|\bar{h}\|}$$

Maximal Ratio Combiner (MRC)

$$= \frac{1}{\sqrt{|h_1|^2 + |h_2|^2}} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$